First Kirchhoff law. A way to solve DC circuits.

We will discuss how to solve simple direct current (DC) circuits. “DC” means that all the voltages and current do not change in time. We know three circuit elements: resistor, ideal voltage source and ideal wire. To enforce an electrical current through a resistor, we must apply to the resistor a finite potential difference (voltage). According to Ohm’s law, the current I is proportional to the voltage U, and the coefficient of proportionality is the resistance R:

$$U = I \cdot R \tag{1}$$

A resistor is usually shown as a zigzag line: 

An ideal voltage source maintains a fixed potential difference between its terminals. This means that if, say, a 3-volt ideal voltage source is connected to points A and B, the voltage difference between the points (potential in point A minus potential in point B) will be 3 volts, no matter what else is connected to them.

An ideal voltage source is shown as: 


An ideal voltage source can accommodate any current. A good way to think about the functioning of an ideal voltage source is that it adjusts its current to maintain fixed voltage between its terminals.

An ideal wire connects the circuit elements. It has zero resistance; hence the current can flow through an ideal wire without a voltage drop across it. If two points of a circuit are connected with an ideal wire, the potential difference between them is zero.

An ideal wire is shown as a solid line: 

Now we will consider how to solve a circuit. Generally, “to solve a circuit” means to find all the currents and voltages. But in most cases, we will have to find just one number: current or voltage across a given element. Such an example is given in Figure below:

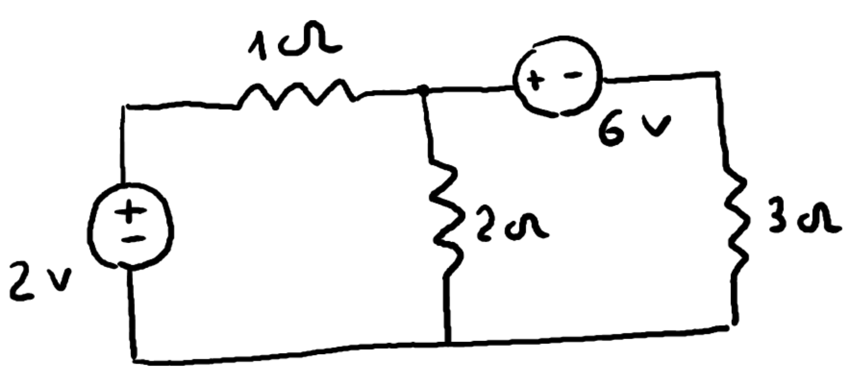



Figure 1.

The circuit contains two voltage sources (we will assume them ideal unless specified otherwise) and 3 resistors. We have to find current through 2 Ohm resistor.

Trying to imagine how the currents will flow is not very productive: here are two voltage sources and the result of their struggle is not clear. Instead, I would use a bit formal, mathematical approach. Let us look at the circuit. It has two special points, which are denoted in Figure 2 below as A and B.

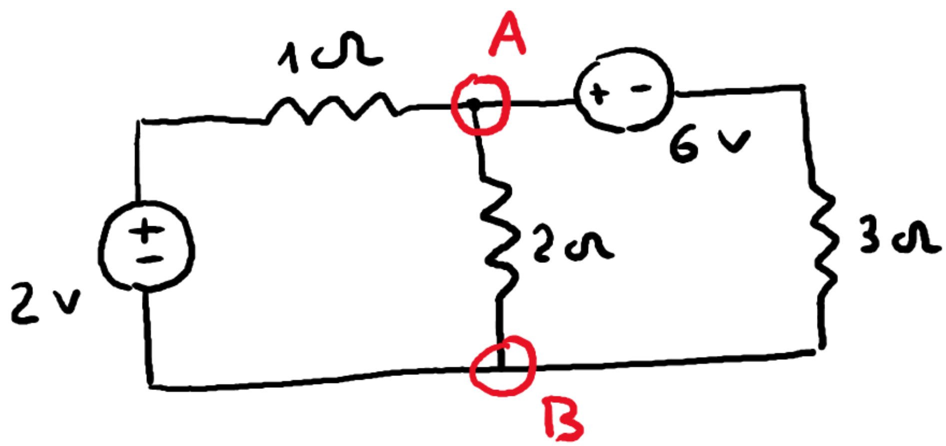



Figure 2.

They are special, because three wires are connected in each of the points. Points of connection of three or more wires are called "nontrivial nodes" or "essential nodes". If we will circle an essential node, three or more wires will cross the circle. The circuit consists of three strings of elements, connected to the essential nodes A and B. In each string, all the elements connected in series. Such strings are called branches. The left branch of the circuit contains 2V source and 1Ohm resistor, the middle one consists of 2Ohm resistor, the right one contains 6V source and 3Ohm resistor. If we would know potentials at points A and B, we could calculate currents in all three branches using Ohm's law. Moreover, since in the *potential difference* (voltage) but not the absolute potentials matters, we can assign a zero potential to any of the essential nodes and treat the potential at the other node as variable we have to find. Let us assign zero potential to node B. Node B then becomes the *reference node*. So the problem is reduced to finding a single number: potential V of node A.

To find a number we need an algebraic equation. Such equation can be obtained from a physical low. The low we are going to use is the charge conservation law. Let us consider the currents which meat at node A. Some of the currents flow into the node, some -out. Charge conservation assumes that in a steady state total net current flowing into the node is equal to zero. Otherwise, the charge in the node would be increasing infinitely which contradicts to charge conservation. The following statement is known as the first Kirchhoff's circuit law:

**Algebraic sum of the currents in conductors meeting at a point is zero**

"Algebraic" here means that the current which flow into the node should have sign opposite to this of the currents flowing out of the node. The law is named after German physicist Gustav Kirchhoff (1824-1887).

So, if we knew the directions of the currents in the branches we could use the first Kirchhoff's law as the equation or, in other words, the *constraint* to find the potential in point A. Now, the beauty of the method we are going to use is that we do not have to know the real current directions. Instead we will just ***choose them arbitrary***! If, in reality, the current flows opposite to the chosen direction it will have negative value. To illustrate the method, I have chosen all three currents flowing into A.

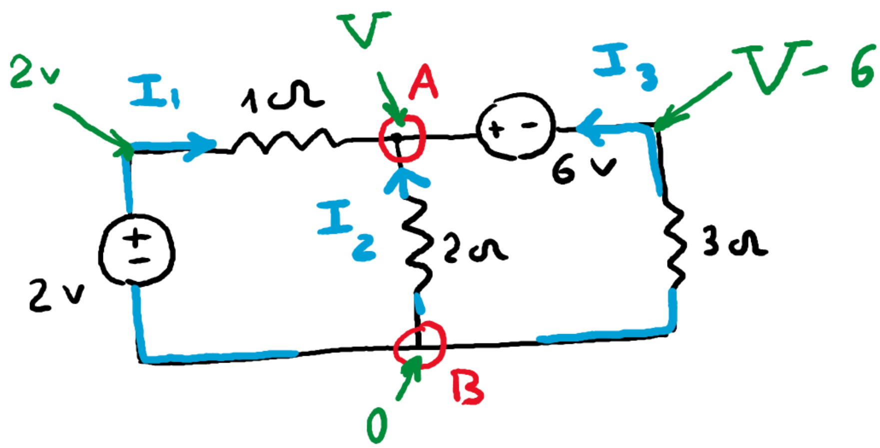



Figure 3.

First Kirchhoff's law gives:

$$I_1 + I_2 + I_3 = 0 \tag{2}$$

Then we will express the currents in the branches using Ohms law and the property of an ideal voltage source. The voltage across an element is calculated according to the chosen current directions: potential in the point *from* where the current flows minus potential in the point *to* where the current flows. Current $I_2$ then is expressed as:

$$I_2 = \frac{0-V}{2Ohm} = -\frac{V}{2} \tag{3}$$

Similarly, we will express the currents $I_1$ and $I_3$. A little difficulty is that we cannot apply Ohm's law to find current through an ideal voltage source – the current can be any. But if the ideal voltage source is connected in series with a resistor, they share the same current. We will use the Ohm's low to find the current through the resistor. The voltage drop across 1Ohm resistor is 2-V. So

$$I_1 = \frac{2-V}{1Ohm} = 2 - V \tag{4}$$

The $I_3$ is:

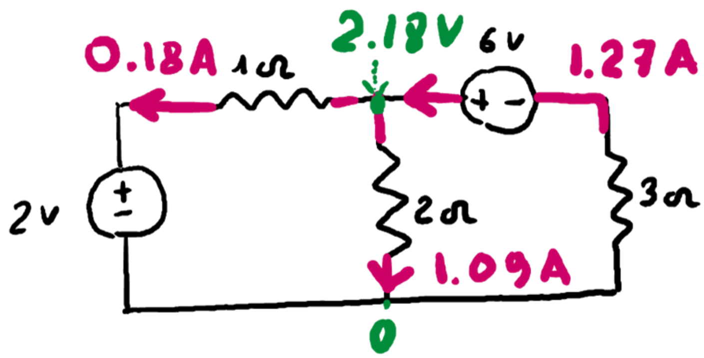
$$I_3 = \frac{0-(V-6)}{3Ohm} = \frac{6-V}{3} \tag{5}$$

Let us plug the expressions for the current into the first Kirchhoff's law (formula 2):

$$\left(-\frac{V}{2}\right) + (2 - V) + \left(\frac{6 - V}{3}\right) = 0$$

$$11V = 24 \Rightarrow V = 2.18V$$

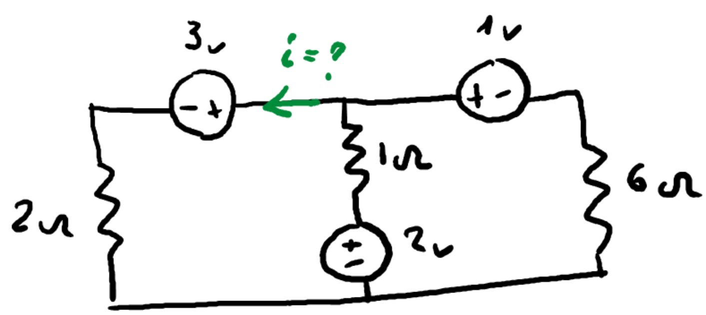
Now we will easily find $I_2$ by plugging 2.18V instead of V into expression(3): $I_2 = -1.09A$ The "minus" sign means that in reality current I2 will flow "downward" rather than "upward" as it is shown in Figure 3. Currents $I_1$ and $I_3$ are -0.18A and 1.27A. The current magnitudes and directions are shown in Figure 4. We can see that as, as expected, total net current flowing into node A is zero.




The method we have discussed is called "nodal analusis".

Problems:

1.Find the current *i*:




2. This one is a bit more challenging. Find potential in point A with respect to this of the reference node: