Thermal motion of electrons.

Remember from last year? All constituents of the substances are constantly moving, we call this type of motion thermal motion and quantity that characterizes it is called temperature. We derived last year that temperature and average kinetic energy of a particle in gas are proportional to each other:

$$\frac{m\overline{v^2}}{2} = \frac{3}{2}kT$$

where $k = 1.38 \times 10^{-23}\frac{J}{K}$ is some coefficient of proportionality called Boltzmann's constant. At room temperature this results in huge thermal velocities – for example for Hydrogen molecules it is about 1754 m/s at 20°C. For water vapor it is 585 m/s. And so on…

Some outer electrons inside some materials can easily fly from one atom to another, they become shared electrons – they are not free, but they are quasi-free – as long as they are inside the solid they behave almost like molecules in the gas, flying where they like. These materials are called conductors. Metals are like that. But now look at the formula above. Do you see that the smaller is the mass the larger the velocity would be? Electron is about 1836 times lighter than proton. So for electron this thermal velocity is above 100 km/s !

Homework problem 1. *Suppose you forgot the value of Boltzmann's k, so you can not directly apply the formula above, but I told you that velocity of hydrogen molecule is 1754 m/s. Can you estimate from this and from the ratio of masses of proton and electron the velocity of electron at the same temperature? Remember, hydrogen molecule has two hydrogen atoms, each of which has one proton and electron flying around it.*

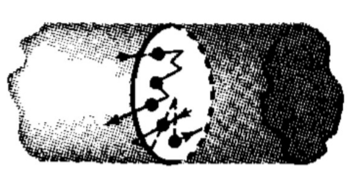
These estimates do not pretend to be exact but give you the feeling about the orders of magnitude. The real theory of electrons in the metal is quantum, it is usually learned after quantum mechanics.

Electric Current.

But if you apply electric field to electrons, this thermal motion overlaps with another kind of motion – ordered drift, caused by electric force.

Ordered, directional motion of charged particles is called ***electric current****.*

Electric current is associated with transporting charges from one place to another. Thermal motion in average does not result in any transport of charge.



People studied currents before the discovery of electrons and historically it is agreed that the direction of current is the direction of motion of positively charge particles, so if the electrons move from left to right, we say that the current flows from right to left.

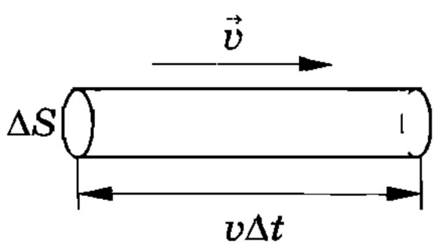
Effects of the Current.

- Thermal
- Chemical
- Magnetic

We do not observe moving of the particles themselves, but we observe the above effects.

Current Density.

Let's consider a current and let's consider a small cylindrical volume



The average velocity

$\vec{v} \ = (\sum_i \vec{v_i}) \ / \ N$

is not zero only because of ordered motion. In time Δt all the particles in the cylinder from the picture will cross the right base of the cylinder. If we denote by n the concentration of charged particles, then the charge crossing that base is

$$\Delta Q = q \ n \ v \ \Delta t \ \Delta S$$

where q is the charge of a single particle.

*A vector whose direction coincides with direction of average velocity and the absolute value is equal to the ratio of the charge transferred across the surface ΔS in time Δt to the ΔS and Δt is called **current density**.*

$$j = \frac{\Delta Q}{\Delta \mathrm{t} \ \Delta S}$$

In our case

$$\vec{j} = q \ n \ \vec{v}$$

The current density is a local quantity. If you add-up charges transferred over time Δt across the whole cross-section of the conductor, you get a quantity which is simply called ***current***

$$I = \frac{\Delta Q}{\Delta \mathrm{t}}$$

Homework problem 2. *Maximal possible current density in copper is* $10^7\ A/m^2$. *Density of copper is* $\rho = 8900\frac{kg}{m^3}$ *and the molar mass of coper is* $M = 0.0635\ kg/mol$ *. Only one electron from each copper atom becomes shared and participates in electron gas. Compute maximal velocity of drift of electrons in copper. Compare it by the order of magnitude to the thermal velocities from problem 1.*

Hint: if N is number of particles in volume V, then concentration is

$$n = \frac{N}{V} = \frac{mN_A}{MV} = \frac{\rho N_A}{M}$$

(because $N = \nu N_A$ – total number of particles is number of moles times number of particles in one mole).

When you solve this homework problem your result will be shockingly low. So low, that it looks like that when you turn on the switch, it should take forever for the light to come up. But we know this is not the case. Why? Because what matters is how fast does the electric field inside the conductor get established.

Ohm Law.

George Simon Ohm

Remember when we spoke about some formulas related to Millikan's droplet experiment, we said that forces of resistance to the motion are frequently proportional to the velocity. We said it for the resistance of air to the droplet. Same happens to the electrons – somehow to keep them moving with the same velocity inside the conductor the electrical force should compensate the resistance force, i.e. should be equal to it. So one may guess that the electrical force, and thus the potential difference U would be proportional to the velocity, and thus to the current I.

Ohm experimentally discovered this proportionality

$$U = R\,I$$

The proportionality coefficient R is called the ***resistance***. The inverse of it is called ***conductance***

$$G = \frac{1}{R}$$

People like to write the Ohm law like this

$$I = \frac{U}{R}$$

which aims to emphasize that "the current appears as a result of potential difference". Our initial formulation was saying "the potential difference needed to maintain the current is equal to $R\,I$ – otherwise the current will decay and disappear". The linear relationship between resistance force and velocity breaks in gas and fluid mechanics for faster velocities – same happens to Ohm law, it has a limited range of applicability, especially in electrolytes and ionized gasses. Metals usually obey the Ohm's law for any voltages for which the metal does not melt.

Resistivity.

It is experimental fact (quite logical though, right?) that

$$R \sim \frac{l}{S}$$

where l is the length and S is the cross-sectional area of the conductor. The coefficient of proportionality ρ is called ***resistivity***

$$R = \rho \frac{l}{S}$$

Homework problem 3. From the dependence above, can you guess what would be total resistance of two similar conductors, each with resistance R, connected sequentially, one after another? How about same two conductors connected parallel to each other (so their left ends are connected and their right ends are connected)?