

Homework for May 13, 2018.

Algebra/Geometry. Complex numbers.

Please, complete the previous homework assignments. Review the classwork handout on complex numbers and complete the exercises. Solve the following problems.

Problems.

1. Find all complex numbers z such that:

a. $z^2 = -i$

b. $z^2 = -2 + 2i\sqrt{3}$

c. $z^3 = i$

Hint: write and solve equations for a, b in $z = a + bi$.

2. On the complex plane, plot all fifth order roots of 1 and all fifth order roots of -1.

3.

a. Find all roots of the polynomial $z + z^2 + z^3 + \dots + z^n$

b. Without doing the long division, show that $1 + z + z^2 + \dots + z^9$ is divisible by $1 + z + z^2 + z^3 + z^4$.

4. Find the roots of the following cubic equations by heuristic guess-and-check factorization, and using the Cardano-Tartaglia formula. Reconcile the two results.

a. $z^3 - 7z + 6 = 0$

b. $z^3 - 21z - 20 = 0$

c. $z^3 - 3z = 0$

d. $z^3 + 3z = 0$

e. $z^3 - \frac{3}{4}z + \frac{1}{4} = 0$

5. Which transformation of the complex plane is defined by:

a. $z \rightarrow iz$

b. $z \rightarrow \left(\frac{1-i}{\sqrt{2}}\right)z$

c. $z \rightarrow (1 + i\sqrt{3})z$

d. $z \rightarrow \frac{z}{1+i}$

e. $z \rightarrow \frac{z+\bar{z}}{2}$

f. $z \rightarrow 1 - 2i + z$

g. $z \rightarrow \frac{z}{|z|}$

h. $z \rightarrow i\bar{z}$

i. $z \rightarrow -\bar{z}$

6. Find the sum of the following trigonometric series using de Moivre formula:

$$S_1 = \cos x + \cos 2x + \cdots + \cos nx = ?$$

$$S_2 = \sin x + \sin 2x + \cdots + \sin nx = ?$$