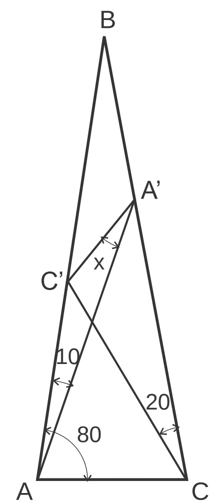
Homework for December 17, 2017.

Geometry.

Review the previous classwork notes. Solve the following problems, including problems from the last homework (if you have not solved them yet).

Problems.

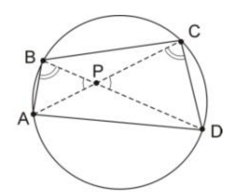
1. In an isosceles triangle ABC with the angles at the base, $\angle BAC = \angle BCA = 80°$, two Cevians CC' and AA' are drawn at an angles $\angle BCC' = 20°$ and $\angle BAA' = 10°$ to the sides, AB and CB, respectively (see Figure). Find the angle $\angle AA'C' = x$ between the Cevian AA' and the segment $A'C'$ connecting the endpoints of these two Cevians.



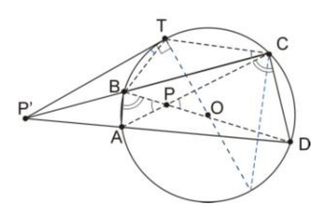
2. Write the proof of the Euclid theorem, which states the following. If two chords AD and BC intersect at a point P' outside the circle, then

$$|P'A||P'D| = |P'B||P'C| = |PT|^2 = d^2 - R^2,$$

where $|PT|$ is a segment tangent to the circle (see Figure).



3. Prove the following Ptolemy's inequality. Given a quadrilateral $ABCD$,



$$|AC| \cdot |BD| \leq |AB| \cdot |CD| + |BC| \cdot |AD|$$

Where the equality occurs if $ABCD$ is inscribable in a circle (try using the triangle inequality).

4. Using the Ptolemy's theorem, prove the following:
    a. Given an equilateral triangle $\Delta\, ABC$ inscribed in a circle and a point Q on the circle, the distance from point Q to the most distant vertex of the triangle is the sum of the distances from the point to the two nearer vertices.
    b. In a regular pentagon, the ratio of the length of a diagonal to the length of a side is the golden ratio, ϕ.

5. Given a circle of radius R, find the length of the sagitta (Latin for arrow) of the arc AB, which is the perpendicular distance CD from the arc's midpoint (C) to the chord AB across it.
6. Prove the Viviani's theorem:

   The sum of distances of a point P inside an equilateral triangle or on one of its sides, from the sides, equals the length of its altitude. Or, alternately,

   From a point P inside (or on a side) of an equilateral triangle ABC drop perpendiculars PP_a, PP_b, PP_c to its sides. The sum $|PP_a| + |PP_b| + |PP_c|$ is independent of P and is equal to any of the triangle's altitudes.

7. *Three Points are taken at random on an infinite plane. Find the chance of their being the vertices of an obtuse-angled Triangle. Hint: use the Viviani's theorem.

Algebra.

Review the last classwork handout. Review and solve the classwork exercises which were not solved and unsolved problems from the previous homeworks.

1. Using Eucleadean algorithm, provide the continued fraction representation for the following numbers. Using the calculator, compare the values obtained by truncating the continued fraction at 1st, 2nd, 3rd, … level with the value of the number itself (in decimal representation).
   a. $\frac{1351}{780}$
   b. $\frac{25344}{8069}$
   c. $\frac{29376}{9347}$
   d. $\frac{6732}{1785}$
   e. $\frac{2187}{2048}$
   f. $\frac{3125}{2401}$
2. Is there a number, x, represented by the following infinite continued fraction? If so, find it.
   a. $x = 5 - \cfrac{6}{5 - \cfrac{6}{5 - \cfrac{6}{5 - \cdots}}}$

b. $x = 2 - \cfrac{1}{2 - \cfrac{1}{2 - \cfrac{1}{2 - \cdots}}}$

c. $x = 1 - \cfrac{6}{1 - \cfrac{6}{1 - \cfrac{6}{1 - \cdots}}}$