Homework for December 10, 2017.

Algebra.

Review the classwork handout. Try to complete solving the unsolved problems from the previous homework. Solve the following problems.

1. Using Eucleadean algorithm, provide the continued fraction representation for the following numbers. Using the calculator, compare the values obtained by truncating the continued fraction at 1st, 2nd, 3rd, … level with the value of the number itself (in decimal representation).
   a. $\frac{1351}{780}$
   b. $\frac{25344}{8069}$
   c. $\frac{29376}{9347}$
   d. $\frac{6732}{1785}$
   e. $\frac{2187}{2048}$
   f. $\frac{3125}{2401}$
2. Is there a number, x, represented by the following infinite continued fraction? If so, find it.
   a. $x = 5 - \frac{6}{5-\frac{6}{5-\frac{6}{5-\cdots}}}$
   b. $x = 2 - \frac{1}{2-\frac{1}{2-\frac{1}{2-\cdots}}}$
   c. $x = 1 - \frac{6}{1-\frac{6}{1-\frac{6}{1-\cdots}}}$

Geometry.

Review the previous classwork notes. Solve the remaining problems from the previous homework. Solve the following problems.

Problems.

1. Prove the following Ptolemy's inequality. Given a quadrilateral $ABCD$,

$$|AC| \cdot |BD| \leq |AB| \cdot |CD| + |BC| \cdot |AD|$$

Where the equality occurs if $ABCD$ is inscribable in a circle.

2. Using the Ptolemy's theorem, prove the following:
   a. Given an equilateral triangle $\triangle ABC$ inscribed in a circle and a point Q on the circle, the distance from point Q to the most distant vertex of the triangle is the sum of the distances from the point to the two nearer vertices.
   b. In a regular pentagon, the ratio of the length of a diagonal to the length of a side is the golden ratio, ϕ.
3. Given a circle of radius R, find the length of the sagitta (Latin for arrow) of the arc AB, which is the perpendicular distance CD from the arc's midpoint (C) to the chord AB across it.
4. Prove the Viviani's theorem:

   The sum of distances of a point P inside an equilateral triangle or on one of its sides, from the sides, equals the length of its altitude. Or, alternately,

   From a point P inside (or on a side) of an equilateral triangle ABC drop perpendiculars PP_a, PP_b, PP_c to its sides. The sum $|PP_a| + |PP_b| + |PP_c|$ is independent of P and is equal to any of the triangle's altitudes.

5. * Three Points are taken at random on an infinite plane. Find the chance of their being the vertices of an obtuse-angled Triangle. Hint: use the Viviani's theorem.