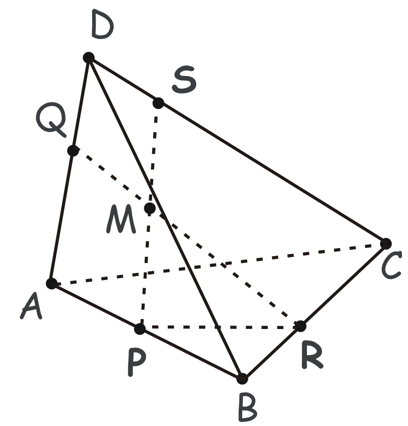
Homework for November 12, 2017.

Geometry.

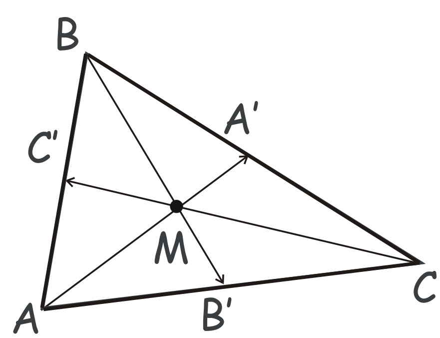
Review the classwork handout. Solve the unsolved problems from previous homework (some are repeated below). Try solving the following problems using the method of point masses and the Law of Lever.

Problems.

1. Prove that if a polygon has several axes of symmetry, they are all concurrent (cross at the same point).
2. Each vertex of the tetrahedron $ABCD$ is connected with the centroid of the opposite face (the crossing point of its medians). Prove that all four of these segments, as well as the segments connecting the midpoints of the opposite edges (opposite edges have no common points; there are three pairs of opposite edges in a tetrahedron, and therefore three such segments) – seven segments in total, have common crossing point (are concurrent).



3. In a quadrilateral $ABCD$, E and F are the mid-points of its diagonals, while O is the point where the midlines (segments conneting the midpoints of the opposite sides) cross. Prove that E, F, and O are collinear (belong to the same line).
4. In a triangle ABC, Cevian segments AA', BB' and CC' are concurrent and cross at a point M (point C' is on the side AB, point B' is on the side AC, and point A' is on the side BC). Given the ratios $\frac{AC'}{C'B} = p$ and $\frac{AB'}{B'C} = q$, find the ratio $\frac{AM}{MA'}$ (express it through p and q).



5. What is the ratio of the two segments into which a line passing through the vertex A and the middle of the median BB' of the triangle ABC divides the median CC'?

6. What is the ratio of the two segments into which a line passing through the vertex A and the middle of the median BB' of the triangle ABC divides the median CC'?
7. In a parallelogram $ABCD$, a line passing through vertex D passes through a point E on the side AB, such that $|AE|$ is $1/n$-th of $|AB|$, n is an integer. At what distance from A, relative to the length, $|AC|$, of the diagonal AC it meets this diagonal?

Algebra.

Review the classwork handout and complete the exercises. Solve the problems from the previous homework, which are repproduced below (skip the ones you have already solved).

1. Find the following sum.

$$\left(2+\frac{1}{2}\right)^2+\left(4+\frac{1}{4}\right)^2+\cdots+\left(2^n+\frac{1}{2^n}\right)^2$$

2. The lengths of the sides of a triangle are three consecutive terms of the geometric series. Is the common ratio of this series, q, larger or smaller than 2?
3. Solve the following equation,

$\frac{x-1}{x}+\frac{x-2}{x}+\frac{x-3}{x}+\cdots+\frac{1}{x}=3$, where x is a positive integer.

4. Find the following sum,
   a. $1+2\cdot 3+3\cdot 7+\cdots+n\cdot(2^n-1)$
   b. $1\cdot 3+3\cdot 9+5\cdot 27+\cdots+(2n-1)\cdot 3^n$
5. Numbers $a_1, a_2, \dots, a_n$ are the consecutive terms of a geometric progression, and the sum of its first n terms is S_n. Show that,

$$S_n=a_1a_n\left(\frac{1}{a_1}+\frac{1}{a_2}+\cdots+\frac{1}{a_n}\right)$$

6. Prove that three terms shown below are the three terms of the geometric progression, and find the sum of its first n terms, beginning with the first one below,

$$\frac{\sqrt{3}+1}{\sqrt{3}-1}+\frac{1}{3-\sqrt{3}}+\frac{1}{6}+\cdots$$

7. What is the maximum value of the expression, $(1+x)^{36}+(1-x)^{36}$ in the interval $|x|\le 1$?
8. Find the coefficient multiplying x^9 after all parenthesis are expanded in the expression, $(1+x)^9+(1+x)^{10}+\cdots+(1+x)^{19}$.