

Math 8: Assignment 16
The Elephant-Hamster Duality
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The Axioms

- **Existence and Association Axiom.** There are elephants and there are hamsters. Elephants exist in families, but the hamsters are all one family. Elephants may be associated to particular hamsters, and hamsters may be associated to particular elephants. Any association is mutual.
- **Unique Association Axiom.** For every two distinct hamsters there is a unique elephant that is associated to both of them. For every two elephants not in the same family, there is a unique hamster associated to both of them. Any two elephants in the same family will never have a common associated hamster.
- **Comprehensive Family Axiom.** For any elephant and any hamster not associated with that elephant, there is a unique elephant in the family that is associated to the given hamster.
- **Theme Song Existence Axiom.** There are theme songs. Theme songs come from groups of elephants associated to a single hamster. Anytime n elephants are associated to a single hamster, there are n corresponding theme songs (repeats of the same song are also allowed). Some example songs: jingle bells, the spongebob theme song (there are many other songs, but we will use these two the most).
- **Magic Machine Axiom.** There is a machine that can combine theme songs. If I want to combine many songs using the machine, the final result does not depend on the order in which I combine the songs.
- **Theme Song Sharpness Axiom.** Anytime two songs are combined, the result is a different song (i.e., the combination cannot be either of the original songs).
- **Spongebob Axiom.** Any set of theme songs, formed by a set of elephants and a single hamster, combine, in total, to the spongebob theme song.
- **Jingle Bells Axiom.** Jingle bells combined with itself gives the spongebob theme song; conversely, jingle bells is the only song that gives the spongebob theme when combined with itself.

Axiom Problems!

Using the above axioms, answer the following questions:

1. Elephants A and B are in the same family. Elephant C is outside the family. If I want to find a hamster associated to both A and C, which axiom do I use?

Consider hamster H is associated to elephants A and C, and hamster J is associated to elephants B and C. Can H and J be the same hamster? Which axiom proves or disproves this?

Unique Association Axiom; also Unique Association Axiom (hamster H cannot be associated to both A and B).

2. Consider elephant A and two hamsters H and J such that neither H nor J is associated to A. Let B be the unique elephant associated to both hamsters H and J. Consider, hypothetically, that elephant B happens to be in the same family as elephant A. If one uses the Comprehensive Family Axiom to find an elephant associated to hamster H that is in the same family as elephant A, must that elephant be elephant B?

Proof: Yes, otherwise hamster H would be associated to both B and another elephant in the family of A and B, which contradicts the Unique Association Axiom.

3. Consider elephants A, B, and C are all associated to hamster H. How many theme songs correspond to this association? Use the Theme Song Existence Axiom.

4. Consider elephants A, B, C, and D are all associated to hamster H: let the corresponding theme songs be called p, q, r, and s. If I combine songs p, q, and r using the Magic Machine Axiom, can the resulting song be the spongebob theme song?

Is it possible for songs p and q to combine to the spongebob theme song?

Is it possible for p to be the spongebob theme song? (Use the Spongebob, Theme Song Sharpness, and Magic Machine Axioms.)

Proof: If one combines a subset of the theme songs corresponding to an association of elephants around the hamster, then one can combine the remaining songs, and these two products would combine to the spongebob theme song, by the Spongebob Axiom. From the Theme Song Sharpness Axiom, we see that neither of these songs can be the spongebob theme song.

5. If elephants A and B are associated to hamster H and the two corresponding theme songs are the same, must it be jingle bells?

Yes, since these songs combine to the spongebob theme by the Spongebob Axiom, we can then use the Jingle Bells Axiom.

6. If elephants A, B, and C are associated to hamster H and the three corresponding theme songs are the same, is it possible for it to be jingle bells?

7. Prove that if n elephants are associated to a single hamster and all the corresponding theme songs are the same, that the theme song can be jingle bells if and only if $n = 2$.