MATH 8
ASSIGNMENT 3: BINOMIAL FORMULA
OCT. 1ST, 2017

Binomial coefficients and binomial formula: a reminder

Recall the numbers

$$ {}_nC_k = \frac{n(n-1)\cdots(n-k+1)}{k(k-1)\cdots 1} = \frac{n!}{(n-k)!k!} \tag{1} $$

These numbers appear in many problems:

$$ \begin{aligned} {}_nC_k = & \text{ The number of ways to choose } k \text{ items out of } n \text{ if the order does not matter} \\ & = \text{The number of words that can be written using } k \text{ zeros and } n-k \text{ ones} \end{aligned} $$

These numbers have one more important application:

$$ (a+b)^n = {}_nC_0 a^n + {}_nC_1 a^{n-1}b^1 + \cdots + {}_nC_n b^n \tag{2} $$

The general term in this formula looks like ${}_nC_k \cdot a^{n-k}b^k$. For example, for $n = 3$ we get

$$ (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 $$

(compare with the 3rd row of Pascal triangle)

This formula is called the **binomial formula**.

Problems

1. Find the coefficient of x^8 in the expansion of $(2x+3)^{14}$

2. Compute $(1+\sqrt{3})^7 + (1-\sqrt{3})^7$

3. Compute $(x+2y)^5 - (x-2y)^5$

4. In how many zeros does the number $11^{100} - 1$ end? [Hint: $11 = 10 + 1$.]

5. It is known that about 20% of all peopel have blue eyes. If you select 10 people at random, what is the probabilty that
(a) All of them have blue eyes
(b) None of them have blue eyes
(c) Exactly half of them have blue eyes.
[Hint: compare with problems about coin tosses from HW1]

***6.** There are 20 boys and 18 girls in a class. How many ways are there to choose 5 pairs for a dance competition? Each pair consists of one boy and one girl; the order of pairs doesn't matter.

7. Finish all unfinished problems from the previous assignments.