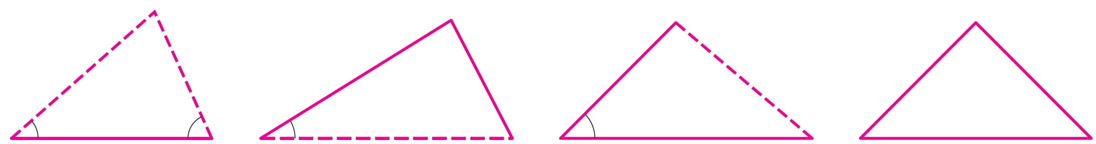
To solve a triangle, we need to know certain information about its sides and angles. A triangle is determined by three of its six parts (angles and sides) as long as at least one of these three parts is a side.



(a) ASA or SAA (b) SSA (c) SAS (d) SSS

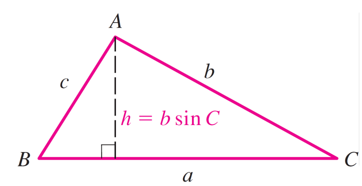
Cases 1 and 2 are solved using the Law of Sines; Cases 3 and 4 require the Law of Cosines.

The Law of Sines

In triangle ABC we have

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

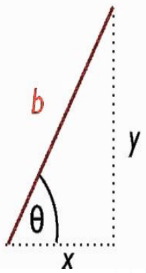
Proof: To see why the Law of Sines is true, refer to the Figure on the right. By the formula in Section 6.3 the area of triangle ABC is $\frac{1}{2}ab\sin C$. By the same formula the area of this triangle is also $\frac{1}{2}ac\sin B$ and $\frac{1}{2}bc\sin A$. Thus,



$$\frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$$

Multiplying by $2/(abc)$ gives the Law of Sines. ■

EXAMPLE: A satellite orbiting the earth passes directly overhead at observation stations in Phoenix and Los Angeles, 340 mi apart. At an instant when the satellite is between these two stations, its angle of elevation is simultaneously observed to be 60° at Phoenix and 75° at Los Angeles. How far is the satellite from Los Angeles?



$$\cos(\theta) = \frac{x}{b}$$
$$\implies x = b\cos(\theta)$$
$$\sin\theta = \frac{y}{b}$$
$$\implies y = b\sin\theta$$

The Law of Cosines

In any triangle *ABC* (see Figure 1), we have

$$a^2 = b^2 + c^2 - 2bc\cos A$$