# Homework 10
# Inequalities

Math 7a

December 6, 2017

In general, we solve polynomial inequalities as follows:

- Find roots of the polynomial: $x_1$, $x_2 \dots$
- Roots $x_1$, $x_2 \dots$ divide the real line into several intervals. We then determine the sign of polynomial in each interval by plugging in an $x$ value called *test point* from each interval.
- If there is a denominator that can equal 0 for some value of $x$, then we include this $x$ among roots used in defining the intervals.
- If the inequalities are strict ($<$ or $>$), then roots themselves are not included in the solution intervals. Otherwise (for $\leq$ or $\geq$), the roots are included in the solution.

**Example**: on which intervals is $(x-4)(x+2) > 0$ true?

- Find roots of the polynomial $f(x) = (x-4)(x+2)$: $x_1 = 4$ and $x_2 = -2$.
- Roots $x_1 = 4$ and $x_2 = -2$ divide the real line into intervals: $\big(-\infty, -2\big)$, $\big(-2, 4\big)$ and $\big(4, +\infty\big)$.
- Picking a *test point* for each of the intervals, say
  - for interval $\big(-\infty, -2\big)$: $x = -3$,
  - for interval $\big(-2, 4\big)$: $x = 0$, and
  - for interval $\big(4, +\infty\big)$: $x = 5$,

  we determine what the sign of our polynomial is on each of the 3 intervals:
  - $f(-3) = (-3-4)(-3+2) = 7 > 0$,
  - $f(0) = (-4)(2) = -8 < 0$,
  - $f(5) = (5-4)(5+2) = 7 > 0$
- Selecting only the intervals where our polynomial takes positive value, we obtain:

$$x \in \big(-\infty, -2\big) \cup \big(4, +\infty\big)$$

- Since the inequality is strict $\Big((x-4)(x+2) > 0\Big)$, the roots $x_1 = 4$ and $x_2 = -2$ are not included in the solution set.

**Problems**

1. Solve the inequalities:
   (a) $x(x-\sqrt{3})(x+\sqrt{5}) \geq 0$

(b) $(x^2 - 1)(x + 3) < 0$

(c) $-3x^2 - x + 4 > 0$

(d) $x^3 + 3x^2 + 3x + 1 \leq 0$

2. Solve the inequalities paying attention to the denominators:

   (a) $\frac{x-1}{x} < 0$

   (b) $\frac{x}{x-1} > 0$

   (c) $\frac{x}{x^2-10} \geq 0$

3. Graph the following equations:

   (a) $x + y = 2$
   *Hint:* rearrange to $y = \ldots$ form.

   (b) $y = x^2$

   (c) $y = x^2 - x$

   (d) $y = x^2 - 5x + 6$