Homework 4
Combination, Pascal's Triangle, Binomial Coefficients

Math 7a

October 18, 2017

						n=0		1						
					n=1		1		1					
				n=2		1		2		1				
			n=3		1		3		3		1			
		n=4		1		4		6		4		1		
	n=5		1		5		10		10		5		1	
n=6		1		6		15		20		15		6		1

Recall that number of ways of choosing k items from a set of n is given by $C_{n,k} = \binom{n}{k} = \frac{n!}{k!(n-k)!}$. We also discussed in class that if probability of success is equal to p, then probability of observing k successes in n attempts/trials is given by: $p(\text{k out of n}) = C_{n,k}p^k(1-p)^{n-k}$.

1. How many "words" of length 5 can one write using only the letters U and R, namely 3 U's and 2 R's? What if you have 5 U's and 3 R's? [Hint: each such "word" can describe a path on the chessboard, U for up and R for right…]

2. How many distinct 4 letter words (doesn't necessarily have to be found in a contemporary dictionary) with exactly 2 vowels and 2 consonants can be constructed if our alphabet consisted of:
   (a) Letters A, O, B, and D?
   (b) All 26 letters of English alphabet? Assume 5 letters are vowels and the rest are consonants.

3. How many "words" of length 5 can one write using only the letters U and R, namely 3 U's and 2 R's? What if you have 5 U's and 3 R's?

4. How many sequences of 0 and 1 of length 12 are there? Sequences of length 12 containing exactly 4 ones? Exactly 8 ones?

5. If we toss a coin 10 times, what is the probability that

(a) all outcomes will be heads?
(b) that there will be exactly one tail?
(c) exactly 2 tails?

6. Let us now assume that our coin is biased: it lands head one third of the time and lands tail two thirds of the time. How likely then is it that 10 coin tosses will result in:
   (a) 2 head? [Hint: head can be counted as a success!]
   (b) more than 2 heads?

7. Let's recall the drunkard problem from class! As usual, our drunkard is walking along a road from the pub to his house, which is located 1 mile north of the pub. Every step he makes can be either to the north, taking him closer to home, or to the south, back to the pub — and it is completely random: every step with can be north of south, with equal chances. After 10 steps for each pair of options, which one is more likely?
   (a) having made 5 steps towards home or 5 steps away from home?
   (b) having made 6 steps towards home or 4 steps towards home?
   (c) having made 2 steps towards home or 7 steps towards home?
   (d) ending up north of the bar or south of the bar?
   (e) returning to the bar or ending up home?