

Classwork 1 and Homework 1

Review of Math 6

Math 7a

September 17, 2017

1. Expand the following expressions.

(a) $2c(x + y)$

(b) $-3x(2x^2 - xy + y^2)$

(c) $(b - 44a)(b + 44a)$

(d) $(3x - 2y)(3x + 2y)$

(e) $(6x^2 - 1)(x^2 - 2x - 4)$

(f) $(a^2 + 2a + 1)(a + 1)$

(g) $(a - b)(a + b)(b^2 - a^2)$

(h) $s(s + 1)(s - 2)(s + 3)$

2. Factorize the following expressions.

(a) $xa - 3xc$

(b) $a^2 + 4a + 4$

(c) $x^3 - x$

(d) $a(b + 3) - c(b + 3)$

(e) $3a^2 + 9a + 2xa + 6x$

(f) $2a^2b + 3ab^2 - 6bx - 4ax$

(g) $a^2 - 11$

(h) $x^3 - xy^2$

(i) $a^4 - b^4$

3. Solve for x . The solution can be a single point, a set of points, an interval, a union of intervals, or an empty set, etc.

(a) $x^2 - 4x + 4 = 0$, for $x \in \mathbb{R}$

(b) $3x^2 - 12 = 0$, for $x \in \mathbb{R}$

(c) $x^2(x - 1)(x^2 - 4) = 0$, for $x \in \mathbb{R}$

(d) $\frac{x+2}{x-2} = 0$, for $x \in \mathbb{R} \setminus \{2\}$

(e) $|x - 3| - 2 = 0$, for $x \in \mathbb{R}$

(f) $|x - 3| + 2 = 0$, for $x \in \mathbb{R}$

(g) $|3x - 1| = 0$, for $x \in \mathbb{R}$

(h) $2x + 1 > 3$, for $x \in \mathbb{R}$

(i) $|2x + 1| - 3 < 2$, for $x \in \mathbb{R}$

(j) $\frac{x+3}{x} > 0$, for $x \in \mathbb{R} \setminus \{0\}$

4. Rationalize the denominator of fraction (multiply by its conjugate).

(a)

$$\frac{3}{3 - \sqrt{5}}$$

(b)

$$\frac{3 - \sqrt{5}}{5 - \sqrt{3}}$$

(c)

$$\frac{1 + \pi}{1 + \sqrt{2}}$$

(d)

$$\frac{3}{3 - \sqrt{5}}$$

5. Compute the following. Simplify your answer if necessary.

(a) $(-1)^{4n-1} + (-1)^{2n-4} + (1)^n$, for $n \in \mathbb{N}$ (a non-negative integer)

(b) $(a + b)^{2c} - (2ab + a^2 + b^2)^c$, for $a, b, c \in \mathbb{N}^*$ (all 3 positive integers)

(c) $4^2 + 2^4 - (1/2)^{-4} - (1/4)^{-2}$

6. Sum each of the following sequences.

(a) 0, 3, 6, ... 99

(b) 1, 2, 4, ... 1024

(c) 3, -3, 3, -3, ... 3

(d) -2, 4, 10, ... 70

7. (a) Assume x_n is a geometric progression where $n \in \mathbb{N}^*$ and $x \in \mathbb{R}$. If $x_{102}/x_{100} = 2$ is true and every pair of consecutive members of the progression have the same sign, what is the constant ratio r of x_n ?

(b) Compute the sum of series $9 + 9^2 + 9^3 + \dots + 9^9$. You may perform addition 8 times, but try to do it in fewer operations.

(c) First member of an arithmetic progression given by $a_n = a_1 + (n - 1)d$ is 1. If sum of first 8 terms exceeds sum of first 4 terms by 70, i.e. $S_8 - S_4 = 70$, what is the common difference d of this progression?

8. If A and B are statements that can be either true or false, prove the following using truth tables.

(a) A **and** B is equivalent to $\neg(\neg A$ **or** $\neg B)$

(b) $A \iff B$ is equivalent to $\neg A \iff \neg B$

9. (a) Which relationship among $>$, $<$ and $=$ holds between

$$\frac{10^{10} - 1}{10^{11} - 1} \text{ and } \frac{10^{11} - 1}{10^{12} - 1}?$$

(b) On what interval is the following true?

$$\frac{x - 1}{x^2 - 1} > \frac{x^2 - 1}{x^3 - 1}$$

10. We throw 10 identical fair dice and observe the resulting numbers. Calculate the probability of each event occurring.

All numbers being 6

(b) At least one of the numbers being 6

(c) At least one of the numbers among the first 3 dice being 6

(d) Out of 10 numbers observed, at least 1 even number being present

(e) What is more likely? i) All numbers being even or ii) at least one number being even?