Math 6a HW21

1. $a = 2^{-13} 3^9$ $b = 2^{11} 3^{-7}$ $ab = (2^{-13} 3^9)(2^{11} 3^{-7})$
$$= 2^{-13+11} \cdot 3^{9-7}$$
$$= 2^{-2} 3^2$$
$$a/b = \frac{(2^{-13} 3^9)}{2^{11} 3^{-7}}$$
$$= 2^{-13-11} \cdot 3^{9-(-7)}$$
$$= 2^{-24} \cdot 3^{16}$$

2. How many trailing zeros? $4^{15} 5^{26}$
→ zeros only made with 10's (2×5)   ⇒ $4^{15} 5^{26} = 2^{2\times15} 5^{26}$
$$= 2^{30} 5^{26}$$
$$= 2^4 \cdot 2^{26} 5^{26}$$
$$= 2^4 \cdot 10^{26} \Rightarrow 26 \text{ zeros}$$

3. Simplify

(a) $\dfrac{3^7 \cdot 2^7}{2^3 \cdot 2^4} = \dfrac{3^7 2^7}{2^7} = 3^7$

(b) $\dfrac{6^5 \cdot 2^4}{3^5 \cdot 2^2} = \dfrac{(2 \cdot 3)^5 \cdot 2^4}{3^5 \cdot 2^2}$
$$= \frac{2^5 \cdot 2^4 \cdot 3^5}{2^2 \cdot 3^5}$$
$$= 2^7$$

(c) $\dfrac{7^9 \cdot 2^5}{7^2 \cdot 2^4} = 7^7 \cdot 2$

(d) $\dfrac{11^4}{11^2 5^2 \cdot 5^3} = \dfrac{11^2}{5^5}$

(e) $\dfrac{7^4 11^2 11^{-5} 7^2 = 7^2}{11^3}$ or $7^2 11^{-3}$

(f) $\dfrac{3^{-5} \cdot 2^7}{3^{-3} \cdot 2^4} = 3^{-2} \cdot 2^3$

(g) $\dfrac{42^2}{6^2} = \dfrac{(6 \cdot 7)^2}{6^2} = 7^2$

(h) $\dfrac{3^5 \cdot 3^{-5}}{3^9} = 3^{-9}$

(i) $\dfrac{x^2 \cdot y^2 \cdot x^{-3}}{x^2} = y^2 x^{-3}$

a_{13} middle term

4. If $a_3 + a_8 + a_{10} + a_{16} + a_{18} + a_{23} = 126$ find the sum of the first 25 terms

$= X$

Take pairs a_3, a_{23} $(3+23=26)$ so sum is $1/3^{rd}$ total   (call it x)

$\qquad\qquad a_8, a_{18}$ $(8+18=26)$ $1/3$

$\qquad\qquad a_{10}, a_{16}$ $(10+16=26)$ $1/3$

$\Rightarrow X = x + x + x = 126$

$\qquad\qquad x = 42 \Rightarrow$ Each pair

$\qquad \Rightarrow$ sum 25 terms $= \dfrac{25 \times 42}{2 \to \text{Pairs}} = 525$

5. $a_1 + a_2 + a_3 = 102$ $a_1 = 15$ find a_{10}

$a_2 = a_1 + d$

$a_3 = a_1 + 2d$

$\quad 102 = a_1 + a_1 + d + a_1 + 2d$

$\quad 102 = 3a_1 + 3d$

$\qquad 34 = a_1 + d$ $a_1 = 15$ $\Rightarrow$ $34 = 15 + d$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \boxed{d = 19}$

$\qquad a_{10} = a_1 + (10-1)d$

$\qquad\qquad = 15 + 9 \times 19$

$\qquad a_{10} = 186$

6. $6 \times a_6 = 9 \times a_9$ what is a_{15}?

$6(a_1 + 5d) = 9(a_1 + 8d)$

$6a_1 + 30d = 9a_1 + 72d$

$\qquad -3a_1 = 42d$

$\qquad\qquad\qquad\qquad\qquad\qquad a_{15} = a_1 + 14d$

$\qquad\qquad\qquad\qquad\qquad\qquad\quad = a_1 + 14\left(\dfrac{-3a_1}{42}\right)$

$\qquad\qquad\qquad\qquad\qquad\qquad\quad = a_1 - a_1 = 0$

7. Find sum 1st 3 elements for

$$a_1 + a_5 = 22$$
$$a_8 - a_5 = 6$$

$$a_8 - a_5 = a_1 + 7d - (a_1 + 4d)$$
$$= 3d = 6 \longrightarrow d = 2$$

$$a_1 + a_5 = a_1 + a_1 + 4d$$
$$= 2a_1 + 8$$
$$= 22$$
$$a_1 = 7$$

$$Sum = a_1 + a_2 + a_3$$
$$= 3a_1 + 3d$$
$$= 3 \times 7 + 3 \times 2$$
$$= 27$$