Math 6b/c: Homework 17
Homework #17 is due February 25.

Counting

We use $|A|$ to denote the number of elements in a set A (if this set is finite). For example, if $A = \{a,b,c,\ldots,z\}$ is the set of all letters of the English alphabet, then $|A| = 26$.

If we have two sets that do not intersect, then $|A \cup B| = |A| + |B|$: if there are 13 girls and 15 boys in the class, then the total is 28.

If the sets do intersect, the rule is more complicated:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Product Rule

If we need to choose a pair of values, and there are *a* ways to choose the first value and *b* ways to choose the second, then there are *ab* ways to choose the pair.

For example, a position on a chessboard is described by a pair like *f4*; there are 8 possible choices for the letter, and 8 possible choices for the digit, so there are $8 \times 8 = 8^2 = 64$ possible positions.

It works similarly for triples, quadruples, … For example, if we toss a coin, there are 2 possible outcomes, heads (H) or tails (T). If we toss a coin 4 times, the result can be written by a sequence of four letters, e.g. HTHH; since there are 2 possibilities for each of the letters, there are $2 \times 2 \times 2 \times 2 = 2^4 = 16$ possible sequences.

Homework

1. Let $A = [1,3] = \{x | 1 \le x \le 3\}$, $B = \{x | x \ge 3\}$, $C = \{x | x \le 1.5\}$. Draw on a number line the following sets (one number line per set):
   (a) $\bar{A}$ (b) $\bar{B}$ (c) $\bar{C}$
   (d) $A \cap B$ (e) $A \cap C$
   (f) $A \cap (B \cup C)$ (g) $A \cap B \cap C$

2. Long ago, in some town a phone number consisted of a letter followed by 3 digits (e.g. K651). How many possible phone numbers could there be? [Note that digits could be zero, i.e. X000 is allowed.]

3. If we roll 3 dice (one red, the other white, and the third black), how many possible combinations are there? How many combinations give the sum of values to be exactly 4?

4. Using a Venn diagram:
   (a) explain why $\overline{A \cap B} = \bar{A} \cup \bar{B}$. Does it remind you of one of the logic laws we had discussed before?
   (b) Do the same for $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5. In this problem, we use $|A|$ to denote the number of elements in a finite set A. We know that for two sets A,B, we have $|A \cup B| = |A| + |B| - |A \cap B|$
   (a) Can you come up with a similar rule for three sets?, that is write a formula for $|A \cup B \cup C|$ which uses $|A|$, $|B|$, $|C|$, $|A \cap B|$, $|A \cap C|$, $|B \cap C|$

6. In a class of 33 students, 12 are girls, 10 play soccer, and 10 play chess. Moreover, it is known that 6 of the soccer players are girls, that 2 of the chess players also play soccer, and that there is exactly one girl who plays both chess and soccer. Finally, 4 girls play neither soccer nor chess. Can you figure out how many boys play soccer, chess, neither, both?