Math 5b: Classwork 21
Homework #21 is due April, 8.

Congruence tests for triangles

By definition, to check that two triangles are congruent, we need to check that corresponding angles are equal and corresponding sides are equal; thus, we need to check 6 equalities. However, it turns out that in fact, we can do it with fewer checks.

Rule 1 (Side-Side-Side rule). If $AB = A'B'$, $BC = B'C'$ and $AC = A'C'$ then $\Delta ABC \cong \Delta A'B'C'$. This rule is commonly referred to as the ***SSS*** rule.

One can also try other ways to define a triangle by three pieces of information, such as a side and 2 angles (ASA), two sides and the angle between them, or three angles and a side (AAA). For the two sides and an angle, there are two versions, one in which the two sides are adjacent to the given angle (SAS) and the other in which one of the given sides is opposite to the given angle (SSA). It turns out that ASA and SAS do indeed define triangles as congruent:

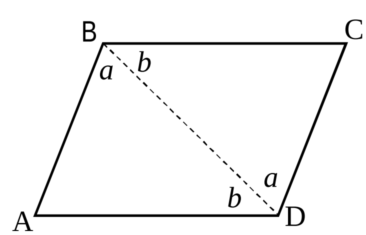
Rule 2 (Angle-Side-Angle Rule). If Ð A = Ð A', Ð B = Ð B' and AB = $A'B'$, then ΔABC≅ $\Delta A'B'C'$. This rule is commonly referred to as ASA rule.

Rule 3 (SAS Rule). If AB = $A'B'$, AC = $A'C'$ and Ð A = Ð A', then ΔABC≅ $\Delta A'B'C'$.

These rules – and congruent triangles in general – are very useful for proving various properties of geometric figures. As an illustration, we prove the following useful result.

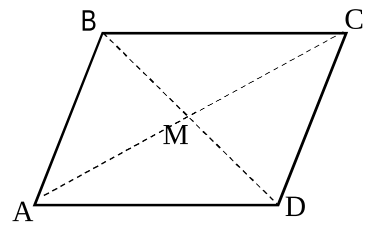
Theorem:
Let $ABCD$ be a *parallelogram*. Then, the <u>opposite sides are equal</u>: $AB = CD$, $AD = BC$.

Proof. Let us draw a diagonal BD. Then the two angles labeled by letter *a* in the figure are equal as alternate interior angle (because AB ∥ DC); also, the two angles labeled by letter *b* are also equal. Thus triangles ΔABD and ΔCDB have a common side, BD, and two angles adjacent to this side which are equal. Thus, by ***ASA*** rule, this two triangles are congruent and the sides $AB = CD$, $AD = BC$.



Homework

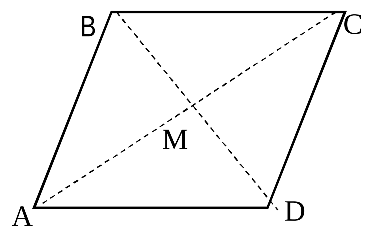
1. Solve the equation $3x + 3 = \frac{1}{2}\,x + 13$
2. (a) Explain why, in a rectangle, opposite sides are equal.
   (b) Show that a diagonal of a rectangle cuts it into two congruent triangles.



3. Let *ABCD* be a parallelogram, and let *M* be the intersection point of the diagonals.

   (a) Show that triangles *ΔAMB* and *ΔCMD* are congruent. [Hint: use the theorem proven in class, that the opposite sides are equal, and ASA.]

   (b) Show that AM = CM, i.e., M is the midpoint of diagonal AC.



4. Let ABCD be a quadrilateral such that sides AB and CD are parallel and equal (but we do not know whether sides AD and BC are parallel).

   (a) Show that triangles *ΔAMB* and *ΔCMD* are congruent.

   (b) Show that sides AD and BC are indeed parallel and therefore *ABCD* is a parallelogram. [Hint: Can you prove that *ΔAMD* and *ΔCMB* are congruent, so that you find equal alternate interior angles?]

5. Calculate using the power rules (power of a product) :

   (a) $3^3 2^3 \left(\frac{1}{6}\right)^3 =$

   (b) $\left(\frac{2}{3}\right)^5 15^5 =$

   (c) $\frac{64^4}{16^4} =$

   (d) $\left(\frac{18}{51}\right)^2 \div \left(\frac{54}{17}\right)^2 =$

   (e) $0.15^3 =$ [Hint: represent as a fraction and then as power of a product]