Math 5b: Classwork 16
Homework #16 is due February 11

Definition: A rational number is a number that can be in the form p/q where p and q are integers and q is not equal to zero.

Example: 2/3 is a rational number because 3 and 2 are both integers

Pigeonhole principle states that if ***n*** items are put **into m** **pigeonholes** with ***n* > *m***, then at least one pigeonhole must contain more than one item.

Theorem: any rational number is a finite or repeating decimal. The way we proved is using Pigeonhole principle.

Review

1. Operations with powers:

$$a^n = a \cdot a \cdot \cdots \cdot a \ (n times)$$

$$(a \cdot b)^n = a^n \cdot b^n$$

$$a^m \cdot a^n = a^{m+n};$$

$$a^m \div a^n = a^{m-n}$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

2. We reviewed solving equations and solving rational equations by multiplying both sides of the equation with the denominator, for example.

$$\frac{(x+1)}{3} = 7$$

$$\frac{(x+1)}{3} \times 3 = 7 \times 3$$

$$(x+1) = 21$$

$$x = 20$$

We also revised the ***identities***:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

And ***factorizing***:

$$a(b+c) = ab + ac$$

… and used them to solve equations.

We solved equations with exponents: $a^x = a^c$ and found out that if we have equal bases we need only compare the exponents (powers) to find the unknown: $x = c$.

So, we need to find a way to rewrite the equations where both sides have the same base.

Homework

1. Solve the following equations for *x*:
   (a) $\frac{5y-12}{3-2y} = 2$

   (b) $\frac{8-2x}{3x-1} = 3$

   (c) $\frac{3x+a}{2a-5x} = -1$

2. Solve the equation:

$$(x-3)^2 - (x-5)(x+5) = 4$$

3. Simplify the fractions using the above identities and factoring rules:
   (a) $\frac{y^2-16}{3y+12} =$

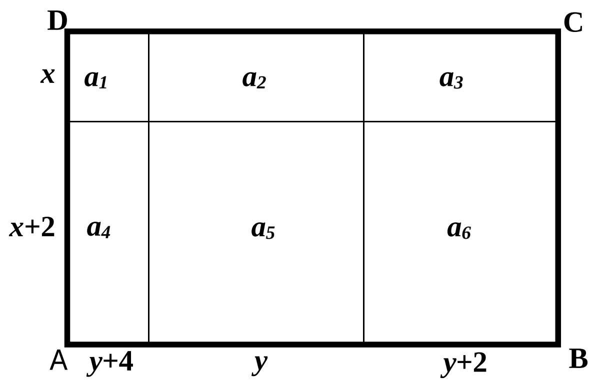
   (b) $\frac{a^2+10a+25}{a^2-25} =$

   (c) $\frac{15z^2-9z}{25z^2-9} =$

4. Consider the sequence 7, 72, 73, . . .7n . . .
   (a) Show that there will be two numbers in this sequence which have the same last two digits. [*Hint: pigeonhole principle!*]

   (b) Show that from some moment, the last two digits of numbers in this sequence will start repeating periodically.

5. ABCD, below, is a rectangle which is split into 6 smaller ones by 3 parallel lines. Find:
(a) The area of each rectangle
(b) The sum of the areas of the 6 rectangles
(c) The total area ABCD
(d) Compare (b) and (c)



6. A worker is earning $24 for each day he works, but he has to pay back $6 for each day he takes off. After 30 days he ended up receiving no money. How many days did he work?

7. Find *n* for
(a) $3^{-n} = 3$
(b) $3^{-n} = 1/3$
(c) $9^{-n} = 81$

8. (*from 101 puzzles in thought and logic, by C. R. Wylie*) Clark, Jones, Morgan, and Smith are four men whose occupation are butcher, druggist, grocer, and policeman, though not necessarily in that order.

Clark and Jones are neighbors and take turns driving each other to work.

Jones makes more money than Morgan.

Clark beats Smith regularly at bowling.

The butcher always walks to work.

The policeman doesn't live near the druggist.

The only time the grocer and the policeman ever meet is when the policeman arrested the grocer for speeding.

The policeman makes more money than the druggist or the grocer.

What is each man's occupation?

9. Written proof of the Pythagorean theorem as discussed in class.