Math 5b: Homework 12
Homework #12 is due January 14.

Review

Powers: $a^n = a \times a \times a \times ... \times a$ (*n* times)

$$a^0 = 1 \qquad \text{read: } a\text{-to-the-zero}$$
$$a^1 = a \qquad \text{is just itself `}a\text{'}$$
$$(ab)^n = a^n \times b^n$$
$$a^n a^m = a^{n+m}$$
$$\frac{a^n}{a^m} = a^{n-m}$$
$$a^n = \frac{1}{a^{-n}} \quad , a^{-n} = \frac{1}{a^n}$$

1. If $a = 2^{19}3^{-91}$ and $b = 2^{-46}3^{-68}$ what is the value of *ab*? of *a/b*?

2. How many zeroes does the number $4^{15}5^{26}$ end with?

Review

<u>Binary Numbers:</u> Numbers represented by using only 0s and 1s.
Powers of 2

n	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
2^n	*1*	*2*	*4*	*8*	*16*	*32*	*64*	*128*	*256*	*512*

Example: Numbers in decimal notation can be presented like this (same as converting a number to a decimal notation): $351 = 3 \times 100 + 5 \times 10 + 1 \times 1$
Similarly, to convert a number into a binary, we need to represent it in powers of 2:
   $351 = \mathbf{256} + 95 = \mathbf{256 + 64} + 31 = \mathbf{256 + 64 + 16} + 15 = \mathbf{256 + 64 + 16 + 8} + 7 = \mathbf{256 + 64 + 16 + 8 + 4 + 2 + 1}$
   $351 = 1{\times}2^8 + 0{\times}2^7 + 1{\times}2^6 + 0{\times}2^5 + 1{\times}2^4 + 1{\times}2^3 + 1{\times}2^2 + 1{\times}2^1 + 1{\times}2^0 = 101011111$

To convert number from binary to decimal we use the familiar rule where we multiply each digit by the position value in base 2. For example:
   $1010 = 1{\times}2^3 + 0{\times}2^2 + 1{\times}2^1 + 0{\times}2^0 = 8 + 0 + 2 + 0 = 10$

<u>*Product formula*</u>: $(x - a)(x + a) = (x^2 - a^2)$

Homework problems

1. Solve the following equations:
   a) $3(x-1) - 4 = 3x + 8$
   b) $\frac{1}{2}(x-1) = -19$
   c) $|2x| = 10$

2. Convert the decimal numbers to binary:
   9, 12, 24, 38, 45

3. Convert the following binary numbers to decimal:
   101, 1001, 10110, 11011, 10101

4. Factor the following number into primes: $99^2 - 9^2$. [Hint: you do not have to compute this number.]

5. Can you find whole numbers a; b such that $a^2 - b^2 = 17$? [Hint: use the formula we talked about in class, and think what $a - b$ and $a + b$ must be.]

6. For the following problem, you need to know that the speed of light is about 300,000 km/sec, and one year is about 3×10^7 seconds.
   a) In astronomy, a common unit of distance is a light year: the distance light covers in one year. How many kilometers is it?
   b) Another common unit of distance in astronomy is a parsec, which is approximately equal to 3×10^{13} km.
      - Can you compute how many parsecs are there in one light year?
      - How many parsecs is the distance between Earth and Sun (The distance is about 1.5×10^8 km)?
      - How many parsecs is the distance between Earth and the Andromeda Nebula (2,000,000,000,000,000,000,000 km)?

7. If $a = 3^{19}5^{-91}$, $b = 2^{-46}3^{-68}$, $c = 2^{54}5^{-8}$, and $d = 10^7 2^{-4}3^{-5}$ what is the value of ab? of a/b? abc? ab/c? $abcd$? ab/cd?

8. How many zeroes does the number $4^{15}5^{26}$ end with?