Classwork 22

April 29 2018

Beginning of Probability.

$$P(A) = \frac{\text{number of outcomes giving A}}{\text{total number of possible outcomes}}$$

- The box contains 10 blue, 10 green, 10 yellow candies. What is the probability to pull 1 green?
- The standard card deck has 4 suits (hearts, diamonds, spades, and clubs); each suit has 13 different card values: 2 through 10, jack, queen, king, and ace. If you randomly draw one card, what is the probability of getting
- A queen od spades?
- A red card?
- A hart card?
- A green card?
- A red queen?

Addition rule

- Suppose we are drawing a card from the deck of 52 cards and ask: what is the probability of getting either queen or king. Since there are 4 queens and 4 kings, which makes it 8 cards total, we can write
- $P(queen\ \boldsymbol{or}\ king) = \frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$
- We can also write it as follows:
- $P(queen\ or\ king) = \frac{4+4}{52} = \frac{4}{52} + \frac{4}{52} = \mathrm{P(queen)} + \mathrm{P(king)}$
- In general, we have the following rule:
- $P(A\ or\ B) = P(A) + P(B)$
- if A and B can't happen together. This rule only applies if A and B **do not** happen together. For example, there are 26 red cards in the deck, so the probability of drawing a red card is $\frac{26}{52} = \frac{1}{2}$. However, if we need to get a red card or a queen, then using the addition formula would give $\frac{26}{52} + \frac{4}{52} = \frac{30}{52}$, **which is incorrect: this way, we have counted red queens twice**. The correct answer is $\frac{28}{52}$: 26 red cards plus two black queens (no need to count red queens, they have already been counted).

Complement rule

- $P(not\ A) = 1 - P(A)$
- For example, probability of drawing a queen from a deck of cards is $\frac{1}{13}$; thus, the probability of drawing something other than a queen is $1 - \frac{1}{13} = \frac{12}{13}$.

- ***Binary numbers***: Details in HW12

  Powers of 2

n	0	1	2	3	4	5	6	7	8	9
$\mathbf{2^n}$	1	2	4	8	16	32	64	128	256	516

Numbers in decimal notation can be presented like this

$351 = 1\cdot2^8 + 0\cdot2^7 + 1\cdot2^6 + 0\cdot2^5 + 1\cdot2^4 + 1\cdot2^3 + 1\cdot2^2 + 1\cdot2^1 + 1\cdot2^0$ =101011111b

Homework

April 29, 2018

1. In the game of roulette, there are 37 slots, numbered 0 through 36. Of numbers 1–36, half are red, the other half are black (zero has no color). What is the probability of hitting
   (a) A number between 1–12 including 1 and 12
   (b) An even number other than zero
   (c) A red number or zero
   (d) If you bet $15 on odd numbers (i.e., you win if you roll one of odd numbers), what is the probability of losing?

2. You roll two dice, one red, and one black. What is the probability of rolling two ones? Of rolling a 4 and a 6?

3. The standard card deck has 4 suits (hearts, diamonds, spades, and clubs); each suit has 13 different card values: 2 through 10, jack, queen, king, and ace. If you randomly draw one card, what is the probability of getting
   (a) The queen of spades
   (b) A face card (i.e., jack, queen, or king)
   (c) A black king
   (d) Anything but the queen of hearts

4. I had drawn a card from the deck, and it turned out to be an ace. Now I am drawing one more card from the same deck. What is the probability that it will be an ace again?

5. Suppose we have a box of 500 candies of different colors and sizes. We know that there are 100 large ones and 400 small ones; we also know that there are 70 red ones, 11 of which are large. From this information, can you compute the probability that a randomly chosen candy will be either red or large? Both red and large?
6. Compute:

$$\frac{2^{1001}3^{999}}{6^{1000}} = 2^{?}3^{?}$$

1. Binary numbers:
   a. Write as binaries: 35, 11, 40
   b. Write as Decimals: 101010b, 11100011b

2. Solve equations:

   a) $|2x + 5| = 1$ b) $\frac{x-4}{x-1} = 3$