MATH 5: Semester 2: Homework 11

JANUARY 7, 2018

Powers of 2

n	0	1	2	3	4	5	6	7	8	9
$\mathbf{2^n}$	1	2	4	8	16	32	64	128	256	516

Yes/no answers

We discussed that if you need to guess a number by asking yes/no questions, you can do it by asking n questions provided that the number of possibilities is at most ; if you need to guess a number between 1–1000, you can do it in 10 questions because **1000 < = 1024**.
The same applies to finding a fake coin among many, and to other similar problems.

Binary numbers

Numbers in decimal notation can be presented like this

$$351 = 3 \cdot 100 + 5 \cdot 10 + 1 \cdot 1$$

Or like this

351=**256** + 95 = **256 + 64** + 31 **= 256 + 64 + 16** + 15 **= 256 + 64 + 16 + 8** + 7 = **256 + 64 + 16 + 8 + 4 + 2 + 1**

351 = $1 \cdot 2^8 + 0 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$ =101011111b

1. Solve the following equations:
   a. 3(x − 1) − 4 = 3x + 8
   b. $\frac{1}{2}(x - 2) = -19$
   c. $|2x| = 10$
   d. $|2x - 5| = 10$

2. Convert the decimal numbers to binary:
   9, 12, 24, 38, 45
3. Convert the following binary numbers to decimal:
   101, 1001, 10110, 11011, 10101
4. You have scales (with two platforms), a 1 gram weight and a large bag of sugar. What would be the fastest way to measure exactly 8 grams of sugar? Exactly 128 grams? Exactly 100 grams?
5. What is the largest number that can be written as a 5-digit binary number ?
6. Is it possible to encode every letter of English alphabet by a 4-digit binary number? You can choose any way you like — for example encoding A as 0000, B as 0001, or by any other method. Would it be possible if we used 5-digit binary numbers? []*It is suggested to think first ,not to go ahead and try all possibilities]*