

## MATH CLUB: PIRATES AND TREASURE!

JAN 14, 2018

1. (This is a repetition of a problem from last year). A detective is working on a murder case. There are 80 people who were present in the area at the time of the murder; one of them is the murderer and one was a witness of the murder. To speed things up, the detective is doing group meetings, inviting to every meeting some of these 80 people. If the witness is in this group and the murderer is not, the witness tells all to the detective. (If the murderer is there, the witness is too scared to talk.).

Can the detective solve the case in 12 meetings?

Can the detective solve the case in 9 meetings?

2. Five pirates are dividing the treasure, consisting of 100 gold coins. The procedure is as follows: the captain suggests a way to divide the coins, then everyone votes. If at least half vote to accept the plan, the treasure is divided as suggested. Otherwise, they kill the captain, make the next pirate in the hierarchy the new captain and start over. (The pirates have a very strict hierarchy - each pirate knows his place in the order of succession to become the captain)

What plan should the pirate suggest to a) survive and b) get as much treasure as he can?

All pirates are very logical and act based on reason only, not emotions. (They are all former professors of mathematics.)

3. There is a well-known way to divide a pile of treasure (which consists of many different kinds of treasure, but all kinds can be divided into arbitrarily small pieces) between two people, each of whom has his own idea of relative value of different pieces so that each believes he got at least half: one person divides the pile into 2 pieces he considers equal, the other chooses one of them.

Can you suggest a generalization which would allow 3 people to divide the treasure so that everyone is happy?

4. Two pirates are dividing the treasure, which consists of two bags of gold coins and a diamond. The first pirate takes some (nonzero) number of coins from one of the bags for himself and also moves equal number of coins from that bag to the other one. After that, the second pirate does the same. This continues as long as it is possible.

The pirate who made the last move also gets the diamond — which both pirates treasure more than all the coins.

If the initial number of coins in the bags was the same, which of the pirates gets the diamond?

What if the number of coins was 100 coins in one bag and 200 in the other?