

School Nova
Challenge Problem
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The Malicious King

A farmer living on the xy -plane wants to fence off a rectangular field with sides parallel to the axes and area at least 1. A malicious king tries to stop the farmer by preemptively placing stakes at infinitely many points in the plane (thereby staking his own claim to those points). It's a constitutional monarchy, so the king can't just claim every point for himself: the law dictates that each point where he places a stake must have a circle of non-zero radius around it that contains no other stakes. (These unclaimed circles around each stake can be of different sizes and can be as small as the king likes.) Can the king place his stakes in such a way that the farmer can't find a rectangle of area 1 with no stakes anywhere in its interior or boundary? (MathCamp Qualifying Quiz 2013)

(Offered prize: cookies/fruits. Bring any solutions or partial solutions to the next club meeting in writing or verbally (either is fine) (and partial credit is given for solution progress).)