MATH CLUB: FIRST MEETING

SEPTEMBER 17, 2017

General Info

In the Problem Solving Club, we will be solving fun (but hard) problems, run math battles, and paricipate in math olympiads such as AMC, Harvard-MIT Math Tournament, and Math Madness. We also give new material — some new theories, or new methods of problem solving — but we will not follow any standard syllabus.

Oh, and we will also be eating pizza!

Some problems

1. In a city of n people, every person has learned some news he wants to share with others. To do so, they start calling each other, Every telephone conversation lasts an hour, and during the conversation, any number of news items can be discussed.

How many hours are necessary for all people to learn all news if

- $n = 128$
- $n = 200$
- $n = 2017$

2. Consider the set of numbers $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$. Is it possible to select 2017 numbers from this set which would form an arithmetic progression?

3. Eleven millipedes are trying to climb the Glass Mountain. The first millipede has 20 legs, the second, 22 legs, and so on, up to the last one that has 40 legs.

The slopes of the mountain are slippery, so to climb it, a millipede must put special climbing shoes on at least half of its feet.

What is the smallest possible number of shoes they need to climb the mountain?

[Small print: initially, all millipedes are down at the foot of the moutnain. At the end, they all must be at the top together. Throwing shoes down the mountain is not allowed. The only way a millipede can carry the shoes up or down the mountain is on its feet — no more than one shoe per foot.]

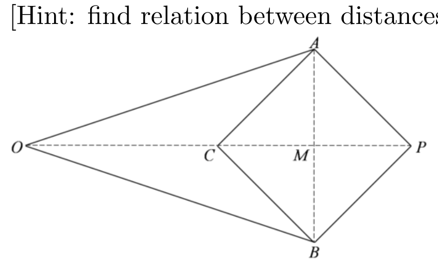
4. Two people are playing a game, moving pieces on a plane. Player one controls a black piece ("wolf"); during his turn, he can move the wolf in any direction by not more than 1cm. Player two controls 100 white pieces ("sheep"); during his turn, he can move **one** of the sheep in any direction by not more than 1cm. Player one starts.

Is it true that no matter what the initial positions were, the wolf will be able to get at least one sheep?

5. The picture below shows a mechanism (Peaucellier Inversor), constructed of rods connected by pivots. (Dashed lines are not part of the mechanism). Point O is fixed (nailed to the plane); all other points can move freely as far as the rods allow. The lengths are such that $OA = OB$, $AC = CB = BP = PA$.

If we move point P in a straight line, what will be the trajectory of point C?

[Hint: find relation between distances OP and OC.]



6. [This is a hard problem — it is from the International Math Olympiad 2017.] A hunter and an invisible rabbit play a game in the Euclidean plane. The rabbit's starting point, A_0, and the hunter's starting point, B_0 are the same. After $n-1$ rounds of the game, the rabbit is at point A_{n-1} and the hunter is at point B_{n-1}. In the n^{th} round of the game, three things occur in order: The rabbit moves invisibly to a point A_n such that the distance between A_{n-1} and A_n is exactly 1. A tracking device reports a point P_n to the hunter. The only guarantee provided by the tracking device to the hunter is that the distance between P_n and A_n is at most 1. The hunter moves visibly to a point B_n such that the distance between B_{n-1} and B_n is exactly 1. Is it always possible, no matter how the rabbit moves, and no matter what points are reported by the tracking device, for the hunter to choose her moves so that after 10^9 rounds, she can ensure that the distance between her and the rabbit is at most 100?